POLYA PROBLEM-SOLVING SEMINAR: THE MASTERCLASS

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1. You and a friend are playing a game against a devil. While your friend is in another room, the devil presents you with a standard (8x8) chessboard with a coin on each square, randomly facing up or down. The devil picks a square and tells you that this square is magic. You then get to choose exactly one coin on the board to turn over. After you flip the coin, your friend is called in and tries to guess which square is the magic square. Can you and your friend devise a strategy beforehand to win this game? You are not allowed to communicate once the game starts. (Brandon Azad)

2. A figure Φ composed of unit squares has the following property: if the squares of an $m \times n$ rectangle (m, n are fixed) are filled with numbers whose sum is positive, the figure Φ can be placed within the rectangle (possibly after being rotated) so that the sum of the covered numbers is also positive. Prove that a number of such figures can be put on the $m \times n$ rectangle so that each square is covered by the same number of figures. (Iurie Bureico)

3. Suppose a country is divided into a bunch of regions. (Country and regions are simply connected, blah blah piecewise differentiable blah.) We say that two regions are neighbors if they share a boundary. Two regions meeting at a vertex only are not neighbors. Show that there is some region with five or fewer neighbors. (Brian Lawrence)

4. A triangulation of a polygon is said to be *admissible* if every internal vertex belongs to at least 6 triangles. Show that there exists some N such that, if a polygon is given an admissible triangulation by N or more triangles, then at least 2,016 of those triangles must share an edge with the polygon. (Brian Lawrence; Putnam 2007A6)

5. Suppose there is a finite (nonempty) collection of people such that every pair of people has exactly one common friend (no one is friends with themself, and friendship is mutual). Show that someone is friends with everybody. (Zeb Brady)

Good luck on Saturday!

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